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On an Eigenplace Function – Mapping Relational to Absolute Space

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Abstract. In this study we provide a descriptive framework of an Eigenplace function that maps from the relational space between objects to the numerical coordinate space that is a part of external reality. Our approach assumes that the Eigenplace function maps cognitively valid space (the relational structure linked with temporal intervals) to a mathematical set of coordinates. After a description of a detailed spatial ontology, some generalizations are discussed.

Keywords: space, time, region, vagueness, motion, Figure and Ground objects.

1. Introduction

What is the location of an object in space? Without examining the definition of object at this point, this is the question underlying the Eigenplace function in its simplest form. Important in this assumption is that, on the one hand, there are relations between objects that we mentally represent (normally by linking them to intervals in time or events), but, on the other hand, these objects are linked with concrete parts or regions of reality that are represented by sets of coordinates.

The Eigenplace function has been applied and examined in different fields ranging from geometry, mathematical models of spatial relations, and Qualitative Spatial Reasoning (Galton, 2000, Galton and Hood, 2005, Hood and Galton, 2006), to human geography (Golledge, 1992) and formal semantics of natural language (e.g., Kracht, 2002, Mador-Haim and Winter, 2015, Piñón, 1993, Pustejovsky, 2013, Wunderlich, 1991, Zwarts and Winter, 2000). However, there are no systematic frameworks converging different perspectives on this function. In the current study we aim at providing a descriptive and cognitively valid and inclusive framework, bringing together most of the current perspectives but also containing an ontology that can be implemented in an axiomatic way (in our case, by using a region-calculus), and corresponding to experimental evidence from research on spatial cognition.

In this paper, we first introduce the idea of the Eigenplace function and describe the constituents of a basic spatial ontology (e.g., regions, paths, and objects) and their main properties. Next, we describe the spatial relations between objects or regions according to the Region Connection Calculus (RCC) formalism extended with some additional operators. Further, the Eigenplace function is defined, and we describe how objects occupying regions in space and time and their relations can be mapped by using this function. This is followed by a description of uses of the Eigenplace function in modeling prepositions and Figure-Ground object roles, expressing them also in relation to the sequential order of time intervals and regions. The final sections of the paper provide remarks about time in Eigenplace functions, the modeling of vagueness and uncertainty in spatial reasoning, and the representation of motion and change of location. We conclude with general observations and a final discussion on the Eigenplace function.

2. Theoretical framework

A spatial configuration can be represented either relationally (which is the way space is cognitively represented) or absolutely (which is the way space is represented mathematically, and corresponds, e.g., to the Cartesian coordinate conception of absolute space). The idea behind the Eigenplace function is that each object in its relational sense is mapped to a concrete position in an absolute space.

An Eigenplace function as such does not explain where an object is, and does not even tell what the adjacent areas and objects are. Rather it presupposes the conception of an absolute space and maps relational information (crucial for spatial cognition) to the absolute (Cartesian or otherwise) space. Although every object has a unique location in absolute space, it is cognitively represented in relation to another object (reference object).

Canonically and informally the Eigenplace function can be defined as follows: every object stands in relation with other objects in a particular time interval and occupies a region in space (Wunderlich and Herweg, 1991, 758). This definition can be extended either by specifying the relations or adding other operators.

In our approach we assume (1) a particular ontology classifying types of constituents and (2) particular sets of relations. Both the ontology and relations – which are further explained below – can be flexibly extended.

2.1. Basic constituents

(cp. Mani and Pustejovsky, 2012, Talmy, 2000, Cohn et al., 1997)

A Basic Spatial Ontology consists of regions, paths, objects, orientation, distance and a few additional perceptual determinants (reference frame, manner and cause of movement). Below we describe these concepts and their main characteristics and principles in more detail. The constituents of the Basic Spatial Ontology are:

a) **regions (places)**;

b) **paths**; all segments of paths are also paths; all segments of paths are subsets or elements of paths but not all paths are segments; also lines are considered as paths;

The basic principles that define paths and places and their mutual structure are as follows:

1. Paths are cognitively represented either as motions ('Mike walks') or as encoding a distinguished part (source, middle part or goal; e.g., 'Jim came from home', 'Black dog ran through the park', 'John went to a movie'). If a path contains a distinguished part, it is perceived *asymmetrically*.
2. *Spatial networks* are generated out of sets of paths (including their intersections) and regions adjacent to them.

c) objects

a. Figures (F; objects to be located);
b. Grounds (G; objects in virtue of which F are located);
c. Viewers (not always involved, and if involved not necessarily a part of the scene; however, location of a viewer (or an observer-induced axis) can determine the perception of Figure and Ground). For more on viewers see after the listing of the components of the ontology (see page 15).

There are several principles that characterize object properties and relations as perceived in places and paths:

1. Figures and Grounds are asymmetric in terms of (1) perceptual and functional prominence and dependencies, (2) their geometrical shape (Ground objects tend to be larger, more stationary; Figure objects – smaller, mobile), and (3) constraints of linguistic encodings (cp. Landau, 1996, Talmy, 2000, Carlson and Covell, 2005).
2. Borders, boundaries, and surfaces are considered as belonging to either regions or objects.
3. Physical objects are spatio-temporally persistent and their paths are connected: every physical object has a unique location and a continuous path in space and time without abrupt jumps, appearances and disappearances in different space or time segments (cp. Gardenfors, 2014, 128f.).
4. Objects are perceptually prior and primary with respect to regions; regions are perceived and discriminated as units of attention in virtue of their objecthood (Figure 1) (Scholl, 2001).
5. Objects are primary with respect to their locations, but the uniqueness of an object is possible in virtue of its location (cp. also Scholl, 2001, 14).

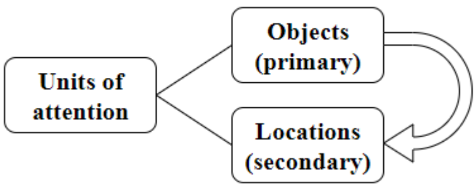

Figure 1. Units of attention (after Scholl, 2001, 14)

6. Events are also considered as objects (serving the role of either Figure or Ground), and only where it is necessary to distinguish between only spatial vs. only temporal readings, objects (in the narrower sense as a part of the spatial domain) can be contrasted with events (as a part of the temporal domain). In both cases, the relation between Figure and Ground holds. In general, the following regularities (Table 1) can be applied (cp. Wunderlich and Herweg, 1991, 760):

Table 1. Localizations and types of Figure (F) relative to a Ground (G) for spatial and temporal objects

Localization of F relative to a G		
Type of object	**F**	**G**
spatial	spatial object (or event)	spatial object
temporal	event (or spatial object)	event

This, however, means that the Eigenplace function would be different for places and events. Instead of an Eigenplace function

$$Eig: O \times T \rightarrow R$$

where O is a set of spatial objects, T – set of time intervals and R – set of spatial regions (places and paths), we would have another version for events

$$Eig_e: \mathcal{E} \times T \rightarrow R$$

where $\mathcal{E}$ is a set of events, T – set of time intervals and R a set of spatial regions (cp. Piñón, 1993). A crucial difference between Eigenplace with objects and Eingenplace with events is that we can never model the precise topology of events because each event has its own topology (if any at all) and eventually several topologies, whereas we can model precise topology of an Eigenplace with spatial objects. (For an alternative view concerning the application of topological relations to cognitive non-spatial relations cp. Lewin, 1936.)[1]

However, even when thinking of spatial objects, it is worth keeping in mind that spatial objects are transformed once they evolve in time (Jiang and Worboys, 2009).

7. In general there are different kinds of objects (cp. Gärdenfors, 2014, 129, Lyons, 1977, 442-445, Van Lambalgen and Hamm, 2005):
   (1) physical and spatial objects: they have a necessarily temporal embeddedness, and their locations are unique and their paths are continuous and connected trajectories; one and the same spatial object cannot be in two different places at the same time,
   (2) temporal objects (e.g., events): they have spatial constituents and they can also have sub-events; events can occur in several places at the same time (e.g., elections),

[1] Another tradition in considering events as objects is Davidsonian semantics (Pustejovsky, 2013, Davidson, 1969, for a different version cp. also Kim, 1973), assuming that the argument structure of a predicate contains a first-order individual *e*, i.e., $P(x_1,,x_n,e)$. Location of an event is a relation between the event variable *e*, and a location argument *l*, i.e., *loc(e,l)*.

(3) abstract objects (e.g., propositions, sets): they do not have direct spatial or temporal properties.

d) **orientation or direction** (determining the relation between Figure and Ground): left, right, under, above.

e) **distance**: near/close, far; here we assume a relational conception of distance consisting of two main operators.

f) **additional determining factors** (neither inherently geometric nor topological):

a. Frames of reference: not a part of a geometrically-topological framework but determining (d) above (the relation between Figure and Gound);
b. Manner of movement;
c. Cause of movement.

In a more general view, physical objects, events, shapes, and indeterminate regions are considered as first-order objects. Further, first-order objects can be classified according to different types – physical, temporal etc.

From a technical point of view, the present framework is largely consistent with Cohn et al., 1997, Randell et al., 1992, Kontchakov et al., 2010, Galton, 2014, in the way that their *sorts in the first-order sorted logic* correspond to the basic primitives in the sense of the present paper, i.e., 'regions' correspond to the sort REGION, 'objects' (Figures and Grounds) correspond to the sort PhysObj. Sort NULL referring to spatially non-existent objects is not represented in the current approach. The further axiomatization is based on the Region Connection Calculus 8 (RCC-8) (Randell et al., 1992) but enriched with several derived and non-derived relations.

2.2. Basic topological and geometric non-functional relations

Spatial objects (regions, paths, objects) are mutually situated in spatial relations that can be characterized topologically or geometrically. Below we describe the basic non-functional topological and geometric relations and their properties. Non-functional relations mean that the differences based on spatial prominence (distinction between Figure and Ground), frequent interaction, experience and general knowledge are not included in this ontology. The basic topological and geometric non-functional relations are:

1. ***Connectedness (C)*** between regions or objects which is the core relation underlying other spatial relations (cp. Cohn et al., 1997, Cohn et al., 1995; for topological interpretations: Galton, 2000, 82f.).[2]

$C(x,y)$: x connects to y

[2] For the predecessor of this conception cp. Randell et al., 1992, Clarke, 1981, Clarke, 1985; for a discussion of the connection relation in context of temporal relations see: Galton 2009. A prominent axiomatic framework assuming connection as a foundational relation is that of B.L. Clarke (1981, 205) arguing that individual variables are spatio-temporal regions bound by two-place predicate 'connected with'. Informal idea about the foundational role of connectedness is also expressed by De Laguna (1922).

Connectedness is
(1) Reflexive: $\forall x[C(x,x)]$;
(2) Symmetric: $\forall x \forall y[C(x,y) \rightarrow C(y,x)]$.

Distance between objects bound by *C(x,y)* is zero.

If using a classical topological representation, we can define regions x and y as connected if their closures have at least one shared point:

$$C(x,y) \equiv_{def} cl(x) \cap cl(y) \neq \emptyset$$

In case of sets in $\mathbb{R}^n$ (sets in an n-dimensional vector space over real numbers) a set *S* is connected if between any two points of *S* there is a continuous path within *S* (Galton, 2000, 147).[3] In a wider sense, the primitive concept in our approach is a connection structure $(\mathcal{R}, C)$ where $\mathcal{R}$ is an arbitrary non-empty set of regions and C a symmetric binary relation on $\mathcal{R}$. The idea of connection structure is based on Whitehead's approach and further developed, made more precise by B.L. Clarke (Gerla, 1995) and enables to define inclusion '$\leq$' such that

$$x \leq y \Leftrightarrow C(x) \subseteq C(y)$$

Further, overlapping '$\mathbf{O}$' is such that

$$x\mathbf{O}y \Leftrightarrow \exists\, z \text{ such that } z \leq x \wedge z \leq y$$

Nontangential inclusion '$\ll$' would mean

$$x \ll y \Leftrightarrow C(x) \subseteq \mathbf{O}(y)$$

such that for every $z \in R$, $\mathbf{O}(z)$ is $C(z)$.

Apart from symmetry of connection relation (A1), the following axioms apply: there is no maximum for $\subseteq$ (A2); for every *x* and *y* there is a *z* that is connected to *x* and *y* (A3); connection is reflexive (A4); $C(x) = C(y) \implies x = y$ (A5); any region *z* contains regions *x* and *y* that are not connected (A6) (Gerla, 1995, 1020, 1022).

According to RCC-8 (Figure 2) (Randell et al., 1992, Mani and Pustejovsky, 2012, 31) and enriched by some further relations (Cohn et al., 1997)[4] the following basic relations derived from *C* can be distinguished:

2. *Disconectedness (DC)*: Regions or objects *A* and *B* do not touch each other; *A* is disconnected from *B*; *DC(A,B)* or formally defined substituting *A* and *B* by the variables *x* and *y*

$$DC(x,y) \equiv_{def} \neg C(x,y)$$

[3] Strictly speaking when two regions are connected within RCC: (a) they share (at least) a common point, or (b) their closures share a common point, or (c) distance between both regions is zero (Dong, 2008, 321, Cohn and Varzi, 2003). A point in RCC can be regarded either as a region or a special case or sort of a region; in the latter case it would be a categorically different object than the region. A more detailed discussion is an issue of another study (but cp. Dong, 2008) but a simple version of the mentioned definitions could be paraphrased by replacing point with region.

[4] For a context cp. also Bennett and Düntsch, 2007, Galton, 2004, Cohn and Renz, 2008, for an extension with Boolean operators cp. Wolter and Zakharyaschev, 2000, Stell, 2000; for a version containing distance and size relations cp. Dong, 2008. Topological and size information is integrated also in the approach by Gerevini and Renz, 2002. Another extension with direction relations is provided by Dube, 2017, Cohn, Li, Liu and Renz, 2014. For a relation-algebraic approach to RCC cp. Düntsch, Wang and McCloskey, 2001.

A topological interpretation of $DC(x,y)$ is $cl(x) \cap cl(y) = \emptyset$ where $cl(x)$ and $cl(y)$ are closed sets.

3. *Part (P)*: A region or object *A* is a part of a region or object *B*; *P(A,B)* or formally defined

$$P(x,y) \equiv_{def} \forall z[C(z,x) \rightarrow C(z,y)]$$

Parthood is
(1) Reflexive: $P(x,x)$,
(2) Transitive: $P(x,y) \wedge P(y,z) \rightarrow P(x,z)$.[5]

A topological interpretation of $P(x,y)$ is $x \subseteq y$.
An inverse version of *P* is also possible

$$Pi(x,y) \equiv_{def} P(y,x)$$

4. *Proper part (PP)*: A region or object *A* is a proper part of a region or object *B* whereby *B* unambiguously includes *A* as its part; *PP(A,B)* or formally defined

$$PP(x,y) \equiv_{def} P(x,y) \wedge \neg P(y,x)$$

A topological interpretation of $PP(x,y)$ is $x \subset y$.
An inverse version of *PP* is also possible

$$PPi(x,y) \equiv_{def} PP(y,x)$$

5. ***Overlap (O)***: A region or object *A* entirely overlaps with a region of object *B*: *O(A,B)* or formally defined

$$O(x,y) \equiv_{def} \exists z[P(z,x) \wedge P(z,y)]$$

A topological interpretation of $O(x,y)$ is $x \cap y \neq \emptyset$.

6. *External connectedness (EC)*: Regions or objects *A* and *B* touch each other at boundaries, i.e., are externally connected; *EC(A,B)* or formally defined

$$EC(x,y) \equiv_{def} C(x,y) \wedge \neg O(x,y)$$

Two regions or objects that touch each other are also called adjacent (Tomko and Winter, 2013, 181).

A topological interpretation of $EC(x,y)$ is $\partial x \cap \partial y \neq \emptyset \wedge x \cap y \neq \emptyset$, where ∂x and ∂y are borders of regions *x* and *y* respectively.

Alternatively if we indicate bounded regions, i.e., their interiors (x^o, y^o), we can define *EC* as $x \cap y \neq \emptyset \wedge x^o \cap y^o = \emptyset$ (cp. also Li and Cohn, 2012).

7. *Partial overlap (PO)*: Regions or objects *A* and *B* partially overlap each other in space; *PO(A,B)* or formally defined

$$PO(x,y) \equiv_{def} O(x,y) \wedge \neg P(x,y) \wedge \neg P(y,x)$$

[5] According to the default interpretation we assume that these formulae are universally quantified; we are omitting universal quantifiers here and elsewhere for the sake of simplicity (cp. also Galton, 2014, 293.)

A topological interpretation of $PO(x,y)$ is $x \cap y \neq \emptyset \wedge x \nsubseteq y \wedge y \nsubseteq x$.

If bounded regions (their interiors are indicated) then *PO* is $x^o \cap y^o \neq \emptyset \wedge x \nsubseteq y \wedge x \nsupseteq y$ (cp. also Li and Cohn, 2012).

8. ***Equality (EQ)***: Regions or objects *A* and *B* occupy the same space (they are spatially identical); *EQ(A,B)* or formally defined

$$EQ(x,y) \equiv_{def} P(x,y) \wedge P(y,x)$$

A topological interpretation of $EQ(x,y)$ is $x = y$.

9. ***Discreteness (DR)***: Regions or objects *A* and *B* are discrete from each other; *DR(A,B)* or formally defined

$$DR(x,y) \equiv_{def} \neg O(x,y)$$

Discreteness can also be expressed as either disconnectedness or external connectedness, i.e.,

$$DR(x,y) \equiv_{def} EC(x,y) \vee DC(x,y)$$

10. ***Tangential proper part (TPP)***: Region or object *A* is inside the region or object *B* and *A* touches the boundary of *B*; *TPP(A,B)* or formally defined

$$TPP(x,y) \equiv_{def} PP(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)]$$

A topological interpretation of $TPP(x,y)$ is $x \subset y \wedge \partial x \cap \partial y \neq \emptyset$.

If boundedness of regions and their interior parts are taken into account we can write $x \subset y \wedge x \not\subset y^o$ where y^o stands for bounded region (cp. also Li and Cohn, 2012).

11. ***Non-tangential proper part (NTPP)***: Region or object *A* is inside the region or object *B* and does not touch the boundary of *B*; *NTPP(A,B)* or formally defined

$$NTPP(x,y) \equiv_{def} PP(x,y) \wedge \neg\exists z[EC(z,x) \wedge EC(z,y)]$$

A topological interpretation of $NTPP(x,y)$ is $x \subset y \wedge \partial x \cap \partial y = \emptyset$. And interpretation where bounded / interior parts are taken into account $x \subset y^o$(cp. also Li and Cohn, 2012).

From *TPP* and *NTPP* directly derived relations that will not be further explored in this paper:

(1) ***Tangential proper part inverse (TPPi)***: Region or object *B* is inside the region or object *A* and *B* touches the boundary of *A*.

(2) **Non-*tangential proper part inverse (NTPPi)***: Region or object *B* is inside the region or object *A* and *B* does not touch the boundary of *A*.

The Basic principles holding for relations 1-11 relate to symmetry._Most of the relations – *C*, *DC*, *DR*, *O*, *PO*, *EC*, *EQ* – are symmetric. The relations *P*, *PP*, *TPP*, and *NTPP* are not symmetric and can have an inverse interpretation (cp. Galton, 2009, 179).

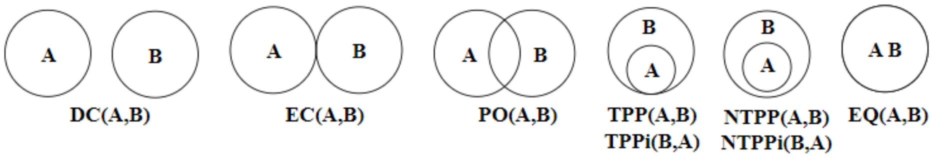

Figure 2. Illustration of RCC-8 relations

The so far described relations are basic topological relations respective to RCC-8. However they can be enriched. A useful relation for expressing everyday contexts is ***convex hull (conv)*** (Cohn et al., 1997, 287ff.; Cohn, 1995; Cohn et al., 1995, a critical discussion cp. Dong, 2008, 349f.).

Convex hull of *x* is a function ***conv(x)*** that can be considered as a spatial primitive[6] referring to the smallest convex region that includes *x*. "A convex region can be defined as one having such a shape that a straight line joining any two points within the region does not go outside it. The *convex hull* of an arbitrary region is then the smallest convex region that contains it[.]" (Cohn et al., 1998, 8)

$$conv(x) \equiv_{def} EQ(x, conv(x))$$

which, in turn, means, e.g., that

$$TPP(x, conv(x));$$
$$P(x, y) \to P(conv(x), conv(y)).$$

The elementary properties of *conv* (cp. Galton, 2000, 182) are as follows*:*

$$x \subseteq conv(x);$$
$$x \subseteq y \to conv(x) \subseteq conv(y);$$
$$conv(x \cap y) \subseteq conv(x) \cap conv(y);$$
$$conv(x) \cup conv(y) \subseteq conv(x \cup y).$$

Conv(x) enables to define regions that are entirely/partly inside or outside the convex hull of *x* but not overlapping *x* (Figure 3) (Cohn et al., 1997, 288, Randell et al., 1992):

(1) inside (*inside*)

$$inside(x, y) \equiv_{def} DR(x, y) \wedge P(x, conv(y))$$

or

$inside(x, y) \equiv_{def} \neg P(x, y) \wedge P(x, conv(y))$ (Cohn et al., 1995, 836)

(2) **partly inside (*p_inside*)**

$$p\_inside(x, y) \equiv_{def} DR(x, y) \wedge PO(x, conv(y))$$

or

$p\_inside(x, y) \equiv_{def} \neg P(x, y) \wedge PO(x, conv(y)) \wedge \exists w[P(w, conv(y)) \wedge \neg P(w, y) \wedge PO(w, x)]$ (Cohn et al., 1995, 836)

(3) **outside (*outside*)**

$$outside(x, y) \equiv_{def} DR(x, conv(y))$$

[6] Thus, the underlying formal theory contains two primitive relations *C*(*x*,*y*) and *conv*(*x*).

or
$outside(x, y) \equiv_{def} \neg P(x, y) \wedge \neg\exists w[P(w, conv(y)) \wedge \neg P(w, y) \wedge PO(w, x)]$ (Cohn et al., 1995, 836).

Also inverse relation of convexity can be formulated.

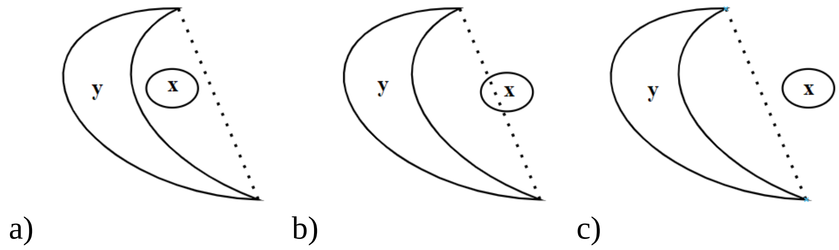

Figure 3. Regions that are a) entirely inside, b) partly inside and c) outside the convex hull

In general, two interpretations of the relation ***inside(x,y)*** should be distinguished: a topological and geometrical (Randell et al., 1992): in the former case (***top_inside***) a region or an object is inside of another region or object if it is a proper part with an surrounding region or object, i.e., there is no cut through the surrounding region or body. In geometrical case (***geo_inside***) an object or a region is inside another but excluding the topological containment.

Accordingly:

$$top\_inside(x, y) \equiv_{def} inside(x, y) \wedge \forall z\big[[conv(z) \wedge C(z, x) \wedge C(z, outside(y)] \rightarrow O(z, y)\big];$$

$$geo\_inside(x, y) \equiv_{def} inside(x, y) \wedge \neg top\_inside(x, y).$$

Thus, topological insideness (containment) has to be distinguished from geometrical insideness that contains convex hull relations as its subsets. In the latter case, relations referring to inside, partial inside and outside have to be distinguished.

Keeping in mind the differences between geometric and topological containment, at least three geometrically and topologically distinct types can be distinguished (Figure 4) (Zwarts, 2017, 14):

(1) **topological enclosure** where regions are related by *TPP* or *NTPP* (and accordingly the inverse relations). E.g., ‘Honey in a closed jar’, ‘A bug in an amber';

(2) **convex geometrical enclosures** where partial geometric enclosure ($p\_inside(x, y)$) is the most prominent instance. E.g., ‘A flower in a vase’;

(3) **scattered geometric enclosure**: enclosure is perceived without any topological connectedness or containment. E.g., ‘Birds in the trees’.

The convexity relation enables to express configurations of inclusion (alternatively they can also be expressed in Wunderlich’s (1993, 124ff.) framework using the so-called focusing effects), where there is a partial inclusion (represented by 'in') of F in G and the verb expresses a supportive or holding function of G, whereby only a part of F is in functional interaction with G and is thus highlighted. But the whole spatial configuration in these cases can be plausibly expressed using convexity relation. Convexity regions frequently contain regions of functional interaction. E.g., ‘The pipe (F) held in the mouth (G)’, ‘The stick (F) held in the hand (G)’.

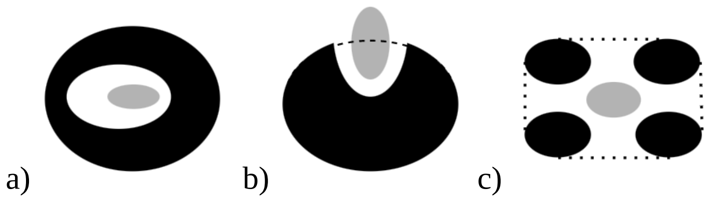

Figure 4. Three types of ‘in’ in RCC: a) total topological enclosure, b) partial geometric enclosure, and c) scattered geometric enclosure (after Zwarts, 2017, 14)

Finally, a relation that can be expressed in applying and combining (1)-(11) relations is **betweenness (*Betw*)**. Object or region B is between A and C if it is also between C and A (Miller and Johnson-Laird, 1976, 61), i.e., $Betw(A, B, C) \leftrightarrow Betw(C, B, A)$.

3. Eigenplace function

As argued before, Eigenplace is a universal and default relation that holds in all spatial relations covering static, locational and dynamic, directional configurations, meaning that all objects (together with temporal segments) are mapped to spatial regions. If the temporal dimension is added then we might define: Every object in time is located in a concrete place (in relation with other objects and within the ontology that we are proposing) and there are no two objects occupying the same place at the same time. If O is a set of arbitrary objects, T is a set of time intervals and R is a set of regions (exact locations in, e.g., a coordinate system), then the simplest versions of the Eigenplace function are:

$$Eig: O \to R$$
$$Eig^t: O \times T \to R$$

Accordingly, an atemporal and a temporal version of Eigenplace have to be distinguished.

The idea of Eigenplace corresponds also to the fundamental principle in geography that there are no two discrete things that occupy the same region in space at the same time (cp. Golledge, 1992, 205).

In what follows we will, first, explore a more restrictive analysis of spatial prepositional relations and their Eigenplace mappings and then move on to a more general approach to Eigenplace relations.

In combining relative and absolute representations we can write

$$Eig: O \to R_{CART}$$

meaning that a set of objects (O) are always located in concrete place (R_{CART}) on a Cartesian plane (although other kinds of coordinate representations are possible).

If we assume that spatial objects are linked to concrete temporal units (*T*), either points or intervals,

$$Eig: O \times T \rightarrow R_{CART}$$

further we assume that

$$r_1, \dots, r_n \in R_{CART}$$
$$o_1, \dots, o_n \in O: O \neq \emptyset$$
$$t_1, \dots, t_n \in T$$

if *Rel* is a subset of spatial relations (RCC extended with some geometric and functional primitives) then

$$Eig: \langle Rel, o_1, \dots, o_n \rangle \times t_i \rightarrow r_j$$

What are canonical options for the ways in which objects can occupy regions in space? According to Galton (2000, 168-170), if *o* is an object and *r* – a region, there are several sets of possible relations:

1. *o* and *r* are congruent (*r* is a possible value of *o*): *DC, EC, PO, EQ*
2. *o* just fits into *r: DC, EC, PO, TPP*
3. *o* covers *r*: *DC, EC, PO, TPPi*
4. *o* can fit right inside *r*: *DC, EC, PO, TPP, NTPP*
5. *o* covers more than *r: DC, EC, PO, TPPi, NTPPi*
6. *o* and *r* are incommensurate (none of above relations holds): *DC, EC, PO*

We might express our framework in terms of Galton (2000) by writing $Rel(Eig(a), Eig(b))$ where *Rel* is a spatial relation and *Eig* are Eigenplace of Figure (*a*) or Ground (*b*). E.g., $DC(Eig(a), Eig(b))$, to denote an object *a* that is outside of another object *b;* further *DC* – disconnectedness relation and *Eig* – the Eigenplace function mapping each object to a concrete place.

In general

$$Rel(Eig(a), Eig(b))$$

where Rel is one of the canonical *RCC* relations (except *EQ*). This means that an object *a* stands in a certain relation to another object *b* if the position of *a* stands in this relation to the position of object *b* (Galton, 2000, 168). In total, it means

$$Rel(Eig(o_1), \dots, Eig(o_n)) \Leftrightarrow Rel(o_1, \dots, o_n) \rightarrow r_j$$

As for the relation *EQ*, we might write

$$EQ(o, r): o \times t \rightarrow r$$

where *o* is an arbitrary object, *t* – a time and *r* – a region (place), which is in turn equivalent to the canonical Eigenplace relation.

To sum up so far, the general schema of the Eigenplace function is

$$\mathcal{R}(o_1, \dots, o_{n:n>1}) \times t_i \rightarrow r_j$$

where $o_1, \dots, o_n$ are arbitrary objects and t_i time intervals, and r_j the corresponding region in real (e.g., Cartesian) space; $\mathcal{R}$ is any arbitrary spatio-temporal relation. To put it more precisely – spatial relations between objects in time occupy a particular region in coordinate space

$$\langle \mathcal{R}, o_1, \dots, o_{n:n>1} \rangle \times T \to R$$

Different spatial relations R_1^s and R_2^s between the same set of objects occur either in the same or different temporal intervals and different locations in coordinate space

$$\langle R_1^s, o_1, \dots, o_{n:n>1} \rangle \times t_i \to r_1$$
$$\langle R_2^s, o_1, \dots, o_{n:n>1} \rangle \times t_j \to r_2$$

It should be kept in mind that spatial relations $\langle R_1^s, \dots, R_n^s \rangle$ are ordered with respect to canonical RCC relations and generalized relations in such a way that $Rel \subset R^S \subseteq \mathcal{R}$.

Another subset of relations is temporal relations R^t such that $R^t \subseteq \mathcal{R}$. We assume that R^toperate on the set of temporal intervals and we also assume that R^t are Allen interval relations (Allen, 1983) such that a temporal structure is $\langle R^t, t_1, \dots, t_{j:j>1} \rangle$.

4. Classical approaches: Eigenplace in prepositions and Figure-Ground roles

In a narrower sense, an Eigenplace function is an intuitively plausible and formally clear approach to the analysis of relations between Figure and Ground (Wunderlich, 1991, 597, Asbury et al., 2008, 12, Zwarts, 1997, Svenonius, 2010, Mador-Haim and Winter, 2015) yielding for every "object or event the place it occupies (its 'Eigenplace'), which is some region" (Wunderlich, 1991, 597). Eigenplace functions are related so that every object is mapped to a particular place in space and all objects are directly or indirectly related in space.

Classically D. Wunderlich provides a *general scheme* holding for all spatial configurations and *particular schemes* describing some specific configurations. A general scheme for prepositional information says that

$$\langle F, G \rangle \in [\![Prep]\!] \text{ iff } Eig[F] \subseteq R_{Prep}[G]$$

where *F* and *G* are Figure and Ground, *Prep* is a spatial preposition or other spatial expression, R_{Prep} is a neighbourhood function for the preposition *Prep*, and Eig is the Eigenplace function.

The important assumption by Wunderlich is that *F*-objects and *G*-objects are paired only indirectly in using the neighbourhood region of *G* (Wunderlich, 1991, 598). The neighborhood region of *G* (*search domain*) is sensitive to perceptual, geometrical, reference frame and functional properties, whereas the properties of *F* determine whether *F* can be included in the neighborhood region of *G*, i.e., $R[G]$.

If a function *INT*[*G*] yields a set of regions *internal* to the Ground, then the 'in' location is

$$\langle F, G \rangle \in [\![in]\!] \text{ iff } Eig[F] \subseteq INT[G]$$

The Eigenplace function complemented with some other basic functions – such as *EXT*[*G*] (referring to regions *external* to *G*), *PROX*[*G*] (referring to regions in the *proximity* of *G*) and additional markers such as *axis-orientation* $\pm VERT$ – allow one to represent other spatial relations as well. E.g.,

$$\langle F, G \rangle \in [\![under]\!] \text{ iff } Eig[F] \subseteq EXT[G, -VERT].$$

Wunderlich also introduces a predicate *LOC(x,r)*, meaning that an object *x* is located in a region *r* and, thus, in fact, fulfills the role of Eigenplace function; and in a more general schema $LOC(F, r[G])$, with the meaning that F is located in the region *r* with respect to *G* (Wunderlich, 1991, 598, Wunderlich, 1993, 113, cp. also Zwarts, 1997, 60f. for a different framework). The basic truth condition here is:

$$LOC(F, r) \text{ is true iff } L[x] \subseteq r$$

where *L* is the Eigenplace of *x* (place occupied by *x*) and '⊆' spatial containment (Wunderlich, 1993, 114).

Practically, this makes it possible to express Eigenplace-relations within a lambda-formalism:

$$\langle F, G\rangle \in [\![under]\!] \text{ iff } Eig[F] \subseteq EXT[G, -VERT] \text{ is identical to}$$
$$\lambda G\lambda F\; LOC(F, EXT[G, -VERT]) \text{ and}$$
$$\langle F, G\rangle \in [\![in]\!] \text{ iff } Eig[F] \subseteq INT[G] \text{ is identical to}$$
$$\lambda G\lambda F\; LOC(F, INT[G])$$

Accordingly, the general scheme is

$$\lambda G\lambda F\; (LOC(F, r[G]) \wedge \mathcal{C}(F, G))$$

where $\mathcal{C}$ refers to additional constraints (e.g., relations of contact, intersection, enclosure) (Wunderlich, 1991, 599, Wunderlich, 1993, 114).

Within this framework the formal representation of basic locational prepositions can be described as follows (Wunderlich, 1993, 113):

'in' $\lambda G\lambda F\; LOC(F, INT[G])$
'by' $\lambda G\lambda F\; LOC(F, EXT[G])$
'over' $\lambda G\lambda F\; LOC(F, EXT[G, +VERT])$
'under' $\lambda G\lambda F\; LOC(F, EXT[G, -VERT])$
'in front of' $\lambda G\lambda F\; LOC(F, EXT[G, +obs])$
'behind' $\lambda G\lambda F\; LOC(F, EXT[G, -obs])$

where '*obs*' means dependence on observer axis: '$+obs$' directed toward the observer and '$-obs$' directed away from the observer.

In some path-expressions Wunderlich adds a dimension parameter *D[F]* that is relative to the movement of the figure on a path and also additional relations – *ENCL* (to enclose *G*), *INTERSEC* (to intersect *G*), to be parallel to the maximal axis of *G* (*PARAL*; *MAX*). The *EXT* and *INT* are defined as $EXT = proximal\_exterior\_of$ and $INT = interior\_of$, and $PROX = EXT \cup INT$ (Wunderlich, 1993, 115-118). These relations allow to model such basic locational prepositions as:

'around' $\lambda G\lambda F\; LOC((F, EXT[G]) \wedge ENCL(D[F], G))$
'through' $\lambda G\lambda F\; LOC((F, INT[G]) \wedge INTERSEC(D[F], G))$
'along'$\lambda G\lambda F\; LOC((F, PROX[G]) \wedge PARAL(D[F], MAX[G]))$

A further important property is that all *objects are located relative to other objects*, i.e., every object has a *neighborhood* of other objects. If *O* is a set of objects, *T* – set of time intervals and *R* – set of regions then there is a family U_j of neighborhood-functions:

$$U_j = \{u_j : O \times T \to R\}, j \in N$$

where $u_j(a,t)$ is a special (concrete) neighborhood of an object a at time *t.* Object b can be localized relative to a according to a neighborhood-function u_j such that

$$p(b,t) \sqsubseteq u_j(a,t)$$

where $\sqsubseteq$ is a spatial part-of-relation. According to $p(b,t) \sqsubseteq u_j(a,t)$ we can say that the place of object b is a part of the j-neighborhood of the object a (cp. Wunderlich and Herweg, 1991, 759, 760). Usually the object b serves the role of focal object (Figure), whereas a is the reference object (Ground) that enables the localization of b, i.e., $p(F,t) \sqsubseteq u_j(G,t)$.

In a more abstract way according to Wunderlich and Herweg (1991, 772f.) we can introduce a general localization relation LOC and, thus, the relations between every two spatial objects can be expressed as either

$$\lambda x \lambda y\ LOC(x, u_j(y)) \text{ or}$$
$$\lambda x \lambda y\ LOC[p(x) \sqsubseteq u_j(y)]$$

where $LOC(x,R)$ is a general localization relation with the meaning that the place of an individual *x* is a spatial part of a spatial region R; U_j is a family of functions u_j assigning certain neighborhoods to individuals and p is a localization function assigning places to individuals. According to Wunderlich and Herweg, the neighborhood relation U_j contains specific differences between spatial relations between objects (in our case, the specific differences are expressed using basic topological and geometric non-functional relations together with orientation and distance primitives. E.g., the meaning of a spatial preposition 'on' can be expressed

$$ON(x,y) \leftrightarrow LOC(x, ON^*(y))$$

where ON^* is a specific neighborhood function characterizing 'on'. More generally

$$\lambda y, \lambda x, LOC(x, ON^*(y))$$

or, using the Figure and Ground distinction,

$$\lambda y, \lambda x, LOC(F, ON^*(G))$$

Thus, the meaning of a spatial preposition is a localization relation between objects (in the case of the current approach, Figure and Ground objects). According to Wunderlich and Herweg (1991, 777), the core pattern of all locative prepositions is the following scheme:

$$\lambda y, \lambda x, LOC(x, PREP^*(y))$$

where *x* is the Figure and *y* the Ground, and $PREP^*$ is a characteristic neighborhood function of a *y* that distinguishes a preposition. E.g.,

$$\lambda y, \lambda x, LOC(x, IN^*(y))$$

is a general and schematic formal representation of the meaning of the preposition 'in'. The background intuition of $LOC(x, IN^*(y))$ is that there is a region $IN^*(y)$ that enables the localization of *x*. I.e., not just the Ground but also a special configurational part (characterized by IN*) of it enables one to locate the Figure. The localization of the Figure (mapped to a spatial region) is enabled only by localization of a Ground (that is also mapped to a spatial region and in this case specified by a particular preposition,

$PREP^*(y)$) (Wunderlich and Herweg, 1991, 777).

A slightly different representation involving observer-induced axis (d) is the case of dimensional prepositions (e.g., describing the area 'in front of') (Wunderlich and Herweg, 1991, 778f.): e.g.,

$$BEVOR(x, y, d) \leftrightarrow LOC(x, BEVOR^*(y, d))$$

Thus, in general, objects – in accordance with the Eigenplace function – are always mapped on spatial regions and are related to each other. In certain cases additional constraints (e.g., based on functional knowledge) have to be applied $\mathcal{C}(x, y)$ (Wunderlich and Herweg, 1991, 777).[7] Such relations can be expressed in the framework of an Eigenplace relation

$$o \times \tau \to \mathscr{r}$$

where o is a type of object, τ – a type of time intervals and $\mathscr{r}$ – a type of spatially extended concrete regions (concrete regions in space). $b_1, \dots, b_n, e_1, \dots, e_n, r_1, \dots, r_n \in o$, where $b_1, \dots, b_n$ is a set of physical objects (e.g., cups, tables, houses), $e_1, \dots, e_n$is a set of events (e.g., birthday celebration, meeting) and $r_1, \dots, r_n$ is the set of regions that are not linked to a concrete spatial area (locationally indeterminate shapes, contours). E.g., 'Celebration party (event) was in the residential area (locationally indeterminate region) in front of the city council building (physical object)'. Paths are also a subset of region types $P_1, \dots, P_n \in \mathscr{r}$ and $t_1, \dots, t_1 \in \tau$.

A more robust formulation (involving also time intervals) of an Eigenplace function (called *Lokalisierungsfunktion p*) is provided by Wunderlich and Herweg (1991, 758): every object in a time interval occupies a region in space:

$$p: O \times T \to R$$

where *O* is a set of objects (whereby also events can be considered as objects; the difference is, however, that in the case of events there are no clear topological relations like in the case of physical objects), *T* – a set of time intervals and *R* – a set of regions and *p* is a certain place. This means that every place (i.e., *p(o,t)*) is a region that is occupied by an object *o* at time *t*.

This formulation of the Eigenplace function corresponds to the *loc'* function by Kracht (2008, 40, 2002, 179, cp. also Piñón, 1993):

$$loc': e \times \tau \to r$$

where e denotes a type of object, τ - type of time-points and r – type of regions. Function *loc'* generates a product of an object and a time point, and returns the region the object occupies at this time.

In expressions of directional spatial relations Eigenplace function refers to the sequential order of times and regions. According to Wunderlich and Herweg (1991), there is a path-function (*Wegfunktion w):*

$$w: O \times SeqT \to SeqR$$

where *O* is a set of objects, *T* – a set of time intervals, *R* – a set of regions, and *Seq* – a sequence relation of time intervals or regions; a, t_i is a region occupied at a certain time

[7] To distinguish from the relation connect, a slightly different symbol is used; initially Wunderlich and Herweg (1991,777) use C(x,y).

t_i, where $0 \leq i \leq 1$. Accordingly a, t_0 is the region at the beginning of a path and $a, t_1$is a region occupied by an object at the end of a path (cp. Wunderlich and Herweg, 1991, 759, for an analysis of Eigenplace functions in vector space semantics cp. Zwarts and Winter, 2000, 175ff.). A general representation of paths in Eigenplace terms (corresponding to the *Wegfunktion w* by Wunderlich and Herrweg, 1991) is:

$$Path\_function: o \times Seq\ \tau \rightarrow Seq\ \mathscr{r}$$

To formalize this idea in terms consistent in the current approach: If *A* and *B* are objects or regions, *REL* is a subset of an extended version of RCC (including additional geometric features that are described before), and τ – a type of time intervals and $\mathscr{r}$ – a type of spatially extended concrete regions (concrete regions in space), then the spatially extended path referred to by *A* and *B* at a certain time is

$$REL(A, B) \times Seq\tau \rightarrow Seq\mathscr{r}$$

such that $\langle t_1, \ldots, t_n \rangle \in \tau$, $(P_1, \ldots, P_n) \in Seq\mathscr{r}$.

5. Remarks on time in Eigenplace

Let us assume time as consisting of intervals.[8] Intervals are linearly ordered and their relations can be constrained as discrete, dense, continuous, bounded or unbounded in each direction (Bennett and Galton, 2004, 16). A general temporal ordering is a History structure

$$\mathcal{H} = \langle S, T, \prec, H \rangle$$

where *S* is a set of states in the world: $s_1, \ldots, s_n$. Further we assume that *S* is a relational structure consisting of at least extended RCC. *T* is a set of time intervals (or points): $t_1, \ldots, t_n$; $\prec$ is irreflexive linear order on *T* (dense, discrete or continuous). *H* is a set of histories $h_1, \ldots, h_n$, i.e., functions from *T* to *S:*

$$H: T \rightarrow S$$

such that $h_1: t_1 \rightarrow s_1$,…, $h_n: t_n \rightarrow s_n$.

We assume some additional functions to describe terminal parts of intervals: $beg(t_i)$ and $end(t_i)$ are functions referring to the beginning and end of an interval t_i.

Further, we agree with Bennett and Galton (2004) and assume a truth functional meaning: $[\![\alpha]\!]_{h,t}^{\mathcal{A}}$, i.e., denotation of expression α at an index $\langle h, t \rangle$ and according to the assignment $\mathcal{A}$ determining the values of non-logical constants: e.g. a set of all assignments for which an expression φ is true, i.e., a truth set *TS* (Bennett and Galton, 2004, 27f.):

$$[\![\varphi]\!]_{TS} = \{\langle \mathcal{A}, h, t \rangle | [\![\varphi]\!]_{h,t}^{\mathcal{A}} = t\}$$

Events consist of intervals and relate to event types. One and the same event type can refer to several events. Intervals $\delta_1, \ldots, \delta_n$ satisfy the event sequence $e_1, \ldots, e_n$ but then $\delta_1, \ldots, \delta_n$ has to satisfy the sequence of event types $e_1^*, \ldots, e_n^*$ such that $e_i^* \subseteq e_i$

[8] We assume that points in both spatial and temporal senses are rather abstractions and special cases than actual parts of the perceivable world, therefore preferring non-atomistic intervals and regions and the basic constituents.

(Bennett and Galton, 2004, 42).

Next we would like to describe the Eigenplace of an event (cp. Pustejovsky, 2013): If an event is a structured object $\mathcal{E}$ where a relation R applies at time t, we can write $\langle R, o_1, \dots, o_n, t\rangle$, then the localization of an object in an event is $loc(o,t) = r_o$. An event with its object localizations is $\langle R, o_1, \dots, o_n, r_{o1}, \dots, r_{on}, t\rangle$, where $r_{o1}, \dots, r_{on}$ are object locations in space.

Normally spatial objects are transformed in time (there is even an approach assuming that events specified via the changes in topological structure are called topological events (Jiang and Worboys, 2009, 34)).

6. Representing vagueness and uncertainty in spatial reasoning

Sometimes we lack the necessary information to determine a spatial location and sometimes spatial objects are inherently vague (e.g., hills, swamps). When dealing with spatial uncertainty or vagueness, it has to be kept in mind that although every spatial object (also vague) has some precise extension in real world (although we do not know it or cannot adequately represent it), we still can use relational information to narrow down the area where the object can be located. We can frequently relationally specify an area where some objects are to be located. This area is a region, or relational structure referring to a concrete extension in the real world.

One way for dealing with vague and uncertain spatial information consistently with RCC-based formalisms is to use an ***anchoring relation***[9] as defined by Galton and Hood (Galton and Hood, 2005, Hood and Galton, 2006, Hood, 2007, for recent applications see: Vasardani et al, 2017, Chen et al., 2017, for an approach in formalization of common sense reasoning of containment in case of incomplete information: Davis et al., 2017; alternative approaches on approximate reasoning in RCC5 and RCC8: Bittner and Stell, 2000).

Anchoring relations enable one to define areas based on *what is known* instead of *specifying a precise location* (which is frequently impossible because of a lack of information). Further, there are at least two ways in which spatial information can be indeterminate: (a) the spatial object we are dealing with might be *vague* (i.e., we cannot define a precise border for it; e.g., hills, forests are instances of spatial objects where they might gradually cease to exist or transform into other spatial objects), (b) spatial information can be *uncertain*, i.e., we might not have enough knowledge to describe the object (see Hood and Galton, 2006, Hood, 2007).

According to this approach we can refer to a known area that in turn includes a region that is indefinite within this area. E.g., we know that an accident occurred in an area where two districts intersect (and if *Distr* stands for a district and *t* for time interval and *r* for a concrete region in real world) then:

$$PO(Distr1, Distr2) \times t_i \rightarrow r_a$$

However, the exact location of the accident $Accident(r_b)$ within r_a is not known. If $r_b \subset r_a$ and if it is known that it occurred somewhere in front of two houses we can write:

$$IN\_FRONT\_OF(Accident, DC(House1, House2)) \times t_i \rightarrow r_b$$

[9] The anchoring relation basically corresponds to the Eigenplace relation.

However, we do not know the exact coordinates of r_b. (This is the reason why we write r_b^V or r_a^V to indicate that r_a or r_b is vague in epistemic terms. I.e.,

$$IN\_FRONT\_OF(Accident, DC(House1, House2)) \times t_i \rightarrow r_b^V)$$

The idea behind the anchoring approach is that there are two different spatial structures:

a. *information space* – information regarding spatial objects, locations and their relations to each other. Information space is expressed in a relational language (i.e., language that is sufficiently rich to allow expressing spatial relations). Information space also contains non-spatial information (e.g., temporal, emotional, social).

b. *precise space* – consisting of exact locations of objects as expressed in a numerical coordinate system (e.g., Cartesian system). Exact space corresponds to an extensional point set topology in a coordinate system.

Information space and precise space are related by mapping information space to precise space in a way that allows *more than one type of relation in information space* to correspond to *one and only one region in precise space*. There are several different ways in which objects in information space can be linked to precise space.

If R is a spatial relation (e.g., one of the extended RCC relations) applying to a set of objects $o_1, \ldots, o_n$, $\mathfrak{C}$ – precise space (e.g., Cartesian coordinate space) and r_k – a region of it (such that $r_k \in \mathfrak{C}$) then

$$R(o_1, \ldots, o_n) \times t_i \rightarrow r_k$$

Possibilities of *anchoring* according to Galton and Hood, 2005; Hood and Galton, 2006; if $o_1, \ldots, o_n \in O$ and $r_k \in \mathfrak{C}$ are:

An object o_i is anchored <u>in</u> r_k means that o_i is located within/inside r_k;

An object o_i is anchored <u>over</u> r_k means that r_k falls within the location of o_i (the location of object o_i contains the whole r_k);

An object o_i is anchored <u>outside</u> r_k means that there is no part of o_i that is located inside of r_k;

An object o_i is anchored <u>alongside</u> r_k means that o_i abuts r_k.

These anchoring relations $\mathbb{A}(O, \mathfrak{C})$ are relating sets of objects (O) in relational space with sets of locations (exact regions) in precise space $\mathfrak{C}$. The intuition behind this is that objects are always located in precise regions even if we do not know exact location.

The idea of anchoring is plausible since we usually talk about objects relationally and use vague and uncertain concepts even though objects do have exact locations (even if we do not know them, which is usually the case). Assuming $r_j, r_k \in \mathfrak{C}$ and $o_i, o_j \in O$, and *loc* is a function denoting the location of an object ($loc \in \mathbb{A}$), we can say according to Hood and Galton (2006) that two constraints apply to anchoring:

(1) if an object is anchored over a region then this region is a part of any region this object is anchored in

$$(in, r_j) \in loc(o_i) \wedge (over, r_k) \in loc(o_i) \rightarrow r_k \subseteq r_j \text{ and}$$

(2) there are no two regions in which an object is anchored such that they are disjoint

$$(in, r_j) \in loc(o_i) \wedge (in, r_k) \in loc(o_i) \rightarrow r_k \cap r_j \neq \emptyset$$

An approach where anchoring is applied to the analysis of preposition 'at' is provided by Vasardani et al., (2017). Imagine the utterance: 'Let us meet at the park'. The meeting point is anchored either (a) inside, (b) along its boundaries, or (c) close to but outside the park. According to Vasardani et al. (2017), a Figure object $F \in o_1, \dots, o_n$ is at Ground object (anchoring area) if and only if F is anchored in the region r_j by Ground object $G \in o_1, \dots, o_n$.

$$R(o_1, \dots, o_n) \times t_i \to r_j$$

Accordingly:
F is in *G* if *F* is anchored in the region by *G*;
F is near *G* if *F* is anchored in the relative complement of *G*.

If $\mathbb{r}$ is anchoring relation $\mathbb{r} \in \{in,\ over, alongside, outside\}$: $\mathbb{r} \subseteq \mathbb{A}$ then anchoring happens as an ordered pair $\langle \mathbb{r}_i, r_j \rangle$ where r is a region in the precise space. Further let us assume that $r_j \in \{r_j^G, r_j^A\}$ where r_j^A means the surrounding area and r_j^G– area occupied by the Ground object. Then we can define (cp. Vasardani, Stirling and Winter, 2017):

$$F\ at\ G \equiv_{def} (in, r_j^A) \in loc(F)$$
$$F\ exactly\_at\ G \equiv_{def} (in, r_j^G) \in loc(F)$$
$$F\ in\ G \equiv_{def} (in, r_j^G) \in loc(F)$$
$$F\ near\ G \equiv_{def} (in, r_j^A - r_j^G) \in loc(F)$$

The regions seem to be mutually nested in the way that $r_j^G \subseteq r_j^A$.

Finally, another approach to vagueness within a region-based formalism is to apply ***tolerance relations*** to RCC relations (Peters and Wasilewski, 2012).

If $x_1, \dots, x_n \in X$ is set of arbitrary spatial entities, *R* set of relations on *X* containing an extended RCC, and if ξ is a set of tolerance relations on *X*, and $t_1, \dots, t_n \in T$ set of temporal intervals, and $c_1, \dots, c_n \in C$ set of concrete locations in physical space then

$$Eig: \langle R, x_1, \dots, x_n, \xi \times t_i \rangle \to c_j$$

When substituting $x_1, \dots, x_n$ with $o_1, \dots, o_n$ we come to a somewhat similar picture to that of anchoring.

7. Representing motion and change of location

Eigenplace can be also modelled when motion is modelled (cp. Lawvere and Schanuel, 2009, 3f.):

$$f_{motion}: time \to space$$

or in more detail we can distinguish between

$$f_1: time \to space$$
$$f_2: space \to line$$
$$f_3: space \to plane$$

According to the composition of the functions we can write:

$$f_a\colon time \rightarrow line$$
$$f_b\colon time \rightarrow plane$$

where line is, e.g., the level of flight and plane is the place occupied by the shadow of a flying object or position of an object located on earth. If for the sake of simplicity we are assuming that *usual* objects are not flying and this feature is left out of consideration for a while, we can write

$$Eig_{motion}\colon \langle R_1(o1, .., o_n) \times t_1, \dots, R_n(o1, .., o_n) \times t_n \rangle \rightarrow r_1, \dots, r_n$$

If o is an object and r is a region (or another object) and $\langle P_1^a, \dots, P_{n:n\geq 1}^a \rangle$ are consecutive segments of a path P^a, then canonically *entering a region* can be modeled as a movement at least with *EC*, *PO*, and *TPP* (for details and additional relations cp. Galton, 2000, 282-284):

$$Mov_{Enteringaregion}\langle EC(o, r, P_1^a), PO(o, r, P_2^a), TPP(o, r, P_3^a) \rangle$$

A crucial component of the process of entering a region is the following regularity

$$Enter(o, r, P_1^a) \rightarrow PO(o, r, P_2^a)$$

where *Enter(o,r)* denotes relation of *o* entering *r*. Informally, when an object enters a region, a part of it is inside and a part is outside of that region (i.e., at least in a certain interval of time the relation between *o* and *r* is *PO*).

Canonical set of possibilities before, during entering, and after entering:

$$\langle \Big(\big(DC(o, r) \vee EC(o, r) \vee PO(o, r)\big) P_1^a \Big), EC(o, r, P_2^a), PO(o, r, P_3^a),$$
$$TPP(o, r, P_4^a), \Big(\big(NTPP(o, r) \vee TPP(o, r) \vee PO(o, r)\big) P_5^a \Big) \rangle$$

Another crucial change of location relationship is *coming into contact*. This can minimally be modelled with *DC* and *EC*:

$$Mov_{coming\ into\ contact}\langle DC(o, r, P_1^a), EC(o, r, P_2^a) \rangle$$

If *o* further enters into *r*, then relation *PO* follows, but this is not necessary the case:

$$\langle DC(o, r, P_1^a), EC(o, r, P_2^a), ((PO(o, r) \vee EC(o, r) \vee DC(o, r))P_3^a) \rangle$$

Another possibility is *movement of an object from one region to another* (Galton, 2000, 286). Two possible cases can be distinguished:

a. Movement starts in the first region and ends in the second (e.g., 'John went from his office to canteen');
b. Starting of the movement contains adjacency and ends with adjacency (e.g. 'Mike went from the chair to the window').

Accordingly:

$$Mov_{from\_to\_containing\_region}\langle TPP(o, r_1, P_1^a), TPP(o, r_2, P_2^a) \rangle$$
$$Mov_{from\_to\_adjacent\_region}\langle EC(o, r_1, P_1^a), EC(o, r_2, P_2^a) \rangle$$

Of course, a movement from one region to another can have the starting point as containing a region and can end with the relation of adjacency (or vice versa).

According to a more recent account (Mador-Haim and Winter, 2015), Eigenplace could be expressed (in a slightly modified way according to the current terminology): a binary relation $\mathbf{far\_from}(\boldsymbol{F}, \boldsymbol{G})$ refers to the following two-place predicate location linking locations (i.e., Eigenplaces) of Figure and Ground – $loc(\boldsymbol{F})$ and $loc(\boldsymbol{G})$. According to Mador-Haim and Winter, Eigenspace (in their terminology) of a Figure is a point (*F*) whereas Eigenspace of a Ground is a region (*G*) (cp. Mador-Haim and Winter, 2015, 442):

$$loc(\boldsymbol{F}) = F$$
$$loc(\boldsymbol{G}) = G$$

This means that the logical form $\mathbf{far\_from}(\boldsymbol{F}, \boldsymbol{G})$ expresses the relation far_from between a point *F* and a region *G*. However, in the current framework this simply means that *F* is a concrete and constrained region whereas *G* is a larger and possibly (although not always) vague or indefinite region (which is the case in far_from).

The core idea by Mador-Haim and Winter (2015) is the Property-Eigenspace Hypothesis (442-443), according to which a relation is between an entity (Figure) and a property (Ground): If $\boldsymbol{F}$ is a Figure and gp a property of the Ground then $\text{far\_from}(loc(\boldsymbol{F}), loc(\text{gp}))$ is far_from relation holding between Eigenplace of Figure and Eigenplace of the properties occupied by Ground. Property-Eigenspace Hypothesis means that every Ground, i.e., property's Eigenspace, "is the union of Eigenspaces for entities in its extension" (Mador-Haim and Winter, 2015, 443). If **gp** is the set of Eigenspaces for properties, then

$$\text{loc(gp)} = \bigcup\{loc(x) : x \in \mathbf{gp}\}$$

Therefore,

$$\text{far\_from}(F, \bigcup\{loc(x) : x \in \mathbf{gp}\})$$

If *F* is a figure and *G* is a Ground and $\mathcal{G}$ is a set of Grounds, then in the framework by Mador-Haim and Winter (2015, 468) some of the core spatial relations can be modelled

$$\text{far\_from}(F, \bigcup \mathcal{G}) \Leftrightarrow \forall G \in \mathcal{G}.\, \text{far\_from}(F, G)$$
$$\text{close\_to}(F, \bigcup \mathcal{G}) \Leftrightarrow \exists G \in \mathcal{G}.\, \text{close\_to}(F, G)$$
$$\text{outside}(F, \bigcup \mathcal{G}) \Leftrightarrow \forall G \in \mathcal{G}.\, \text{outside}(F, G)$$
$$\text{inside}(F, \bigcup \mathcal{G}) \Leftrightarrow \exists G \in \mathcal{G}.\, \text{inside}(F, G)$$

Consistently with their approach (Mador-Haim and Winter, 2015, 472f.) we can model part-whole relations: if F is a subpart of G, then $loc(F) \subseteq loc(G)$. Therefore, if the regions or elements in the set $\mathcal{F}$ are subparts of *G*, then $\bigcup_{F \in \mathcal{F}} loc(F) \subseteq loc(G)$.
According to our framework

$$\langle far\_from, F, G \rangle \times t_i \to r_n.$$

8. Conclusion

The Eigenplace relation covers the core of the processes occurring when mapping relational spatial and temporal information to a coordinate space. This mapping is an essential step once cognitive structures (operating in relational spatio-temporal space) are linked with mathematical coordinate structures operating in numerical terms outside of the human mind.

In our approach, we have defined an ontology that can be used in applications of Eigenplace to resolve spatial vagueness in static and dynamic terms (covering simple and more complex types of movement and motion in space). This corresponds to the idea that the trajectory of an object in space is always linked to a function in time (i.e., there are no spatial movements lacking temporal correlates).

A particularly important direction in our approach is to map vague relational space and accurate space by using the spatial anchoring relation (Galton and Hood, 2005, Hood and Galton, 2006). The anchoring relation is central in spatial communication in general and spatial dialogue systems in particular. Although, cognitively, spatio-temporal existence of objects is always relational and can be cognitively represented in vague or uncertain ways, in virtue of anchoring they can be mapped toprecise coordinate space (i.e., relational objects have exact numerical coordinate correlates), in principle independently of whether we know them or not.

Our developed spatio-temporal ontology operates in an extended RCC formalism (Cohn et al., 1997) and is flexible and open to potentially include other constituents and operators (for functional extensions see also Šķilters et al., 2024). Based on the operator of connectedness (a core operator from which the majority of other operators can be derived) we are able to describe most of the geometrically, topologically, and functionally crucial operators that operate in everyday environments. The most important is the functional operator of locational control, binding the figure and ground object according to the principle that, once the ground is moved in space / time, the figure is moved as well. In these cases, containment is perceived even if it does not apply in the topological sense.

An underlying principle in our approach is the functional prominence of spatio-temporal objects assuming the asymmetry of central (Figure) object and reference (Ground) object that operates in spatial, temporal, and spatio-temporal settings. In the case of temporal situations we are dealing with events as the objects.

Our results can be applied for natural language contexts (especially for modeling the semantics of spatial expressions) but are also usable for non-linguistic spatial information. Although some parts of our approach have been experimentally tested (e.g., Žilinskaitė-Šinkūnienė et al., 2019, Zariņa et al., 2023), there are several spatio-temporal relations (e.g., type of movement and motion, topological features of temporal objects) that can still be both experimentally and computationally tested.

Abbreviations

F – Figure object
G – Ground object
RCC-8 – Region Connection Calculus 8

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